3-category

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次元がもう一つ上がり，2-morphism の間の射も存在するのが 3-category である．即ち

定義. (**Cat**-**Cat**)-豊穣圏を strict 3-category という．

3-category の場合も weak バージョンがあり，それを tricategory という．定義の前に一つ注意をしておく．$\mathcal{B}, \mathcal{C}$ を bicategory，$F, G\colon \mathcal{B} \longrightarrow \mathcal{C}$ を pseudofunctor とすると $\mathrm{Fun}_{\mathrm{ps}}(\mathcal{B}, \mathcal{C})$ は bicategory である．よって $F, G \in \mathrm{Fun}_{\mathrm{ps}}(\mathcal{B}, \mathcal{C})$ の間の adjoint equivalence $\sigma \dashv \sigma^*\colon F \Longrightarrow G$ を考えることができる．以下ではこのように, adjoint equivalence に対してその右随伴を $*$ を付けて表す.

定義. tricategory $\mathcal{T}$ とは，以下を満たすことをいう．

(1) 集まり $\mathrm{Ob}(\mathcal{T})$ が与えられている．$\mathrm{Ob}(\mathcal{T})$ の元を対象 (もしくは 0-cell) と呼ぶ.

(2) 各対象 $a, b \in \mathcal{T}$ に対して bicategory $\mathcal{T}(a, b)$ が与えられている．($\mathcal{T}(a, b)$ を a と b の hom-bicategory と呼ぶ．)

(3) 各対象 $a, b, c \in \mathcal{T}$ に対して pseudofunctor $C^{abc}\colon \mathcal{T}(b, c) \times \mathcal{T}(a, b) \longrightarrow \mathcal{T}(a, c)$ が与えられている.

(4) 各対象 $a \in \mathcal{T}$ に対して pseudofunctor $I^a\colon \mathbf{1} \longrightarrow \mathcal{T}(a, a)$ が与えられている．

(5) 各対象 $a, b, c, d \in \mathcal{T}$ に対して次の adjoint equivalence α^{abcd} が与えられている．

$$\begin{tikzcd}
 & \mathcal{T}(c,d)\times\mathcal{T}(b,c)\times\mathcal{T}(a,b) \arrow[dl, "C^{bcd}\times\mathrm{id}"'] \arrow[dr, "\mathrm{id}\times C^{abc}"] & \\
\mathcal{T}(b,d)\times\mathcal{T}(a,b) \arrow[dr, "C^{abd}"'] \arrow[rr, phantom, "\underset{\alpha^{abcd}}{\Longrightarrow}"] & & \mathcal{T}(c,d)\times\mathcal{T}(a,c) \arrow[dl, "C^{abd}"] \\
 & \mathcal{T}(a,d) &
\end{tikzcd}$$

(6) 各対象 $a,b\in\mathcal{T}$ に対して次の adjoint equivalence λ^{ab},ρ^{ab} が与えられている．

$$
\begin{tikzcd}
\mathbf{1}\times\mathcal{T}(a,b) \arrow[rr, "\cong"] \arrow[dr, "I^b\times\mathrm{id}"'] & {} \arrow[d, phantom, "\lambda^{ab}\Uparrow"] & \mathcal{T}(a,b) \\
& \mathcal{T}(b,b)\times\mathcal{T}(a,b) \arrow[ur, "C^{abb}"'] &
\end{tikzcd}
\qquad
\begin{tikzcd}
\mathcal{T}(a,b)\times\mathbf{1} \arrow[rr, "\cong"] \arrow[dr, "\mathrm{id}\times I^a"'] & {} \arrow[d, phantom, "\rho^{ab}\Uparrow"] & \mathcal{T}(a,b) \\
& \mathcal{T}(a,b)\times\mathcal{T}(a,a) \arrow[ur, "C^{aab}"'] &
\end{tikzcd}
$$

(7) 各対象 $a,b,c,d,e\in\mathcal{T}$ に対して，次の同型な modification $\mathfrak{A}^{abcde}$ が与えられている．(ここで $\mathcal{T}cba := \mathcal{T}(b,c)\otimes\mathcal{T}(a,b)$ などの略記を行った)

$$
\begin{tikzcd}[column sep=small]
& & \mathcal{T}edcba \arrow[dll, "C^{cde}\times\mathrm{id}\times\mathrm{id}"'] \arrow[drr, "\mathrm{id}\times\mathrm{id}\times C^{abc}"] \arrow[dd, "\mathrm{id}\times C^{bcd}\times\mathrm{id}"] & & \\
\mathcal{T}ecba \arrow[dd, "C^{bce}\times\mathrm{id}"'] & & & & \mathcal{T}edca \arrow[dd, "\mathrm{id}\times C^{acd}"] \\
{} \arrow[r, phantom, "\overset{\alpha^{bcde}\times\mathrm{id}}{\Longrightarrow}"] & {} & \mathcal{T}edba \arrow[dll, "C^{bde}\times\mathrm{id}"] \arrow[drr, "\mathrm{id}\times C^{abd}"'] & {} & {} \arrow[l, phantom, "\overset{\mathrm{id}\times\alpha^{abcd}}{\Longrightarrow}"] \\
\mathcal{T}eba \arrow[drr, "C^{abe}"'] & & {} \arrow[d, phantom, "\overset{\alpha^{abde}}{\Longrightarrow}"] & & \mathcal{T}eda \arrow[dll, "C^{ade}"] \\
& & \mathcal{T}ea & &
\end{tikzcd}
\quad \overset{\mathfrak{A}^{abcde}}{\underset{\sim}{\Rrightarrow}} \quad
\begin{tikzcd}[column sep=small]
& & \mathcal{T}edcba \arrow[dll, "C^{cde}\times\mathrm{id}\times\mathrm{id}"'] \arrow[drr, "\mathrm{id}\times\mathrm{id}\times C^{abc}"] & & \\
\mathcal{T}ecba \arrow[dd, "C^{bce}\times\mathrm{id}"'] \arrow[dr, "\mathrm{id}\times C^{abc}"] & & = & & \mathcal{T}edca \arrow[dd, "\mathrm{id}\times C^{acd}"] \arrow[dl, "C^{cde}\times\mathrm{id}"'] \\
{} \arrow[r, phantom, "\overset{\alpha^{abce}}{\Longrightarrow}"] & \mathcal{T}ace \arrow[dd, "C^{ace}"] & & \mathcal{T}ace & {} \arrow[l, phantom, "\overset{\alpha^{acde}}{\Longrightarrow}"] \\
\mathcal{T}eba \arrow[dr, "C^{abe}"'] & & & & \mathcal{T}eda \arrow[dl, "C^{ade}"] \\
& \mathcal{T}ea & & &
\end{tikzcd}
$$

(8) 各対象 $a,b,c\in\mathcal{T}$ に対して，次の同型な modification $\mathfrak{B}^{abc},\mathfrak{L}^{abc},\mathfrak{R}^{abc}$ が与えられている．

$$
\begin{tikzcd}[column sep=small]
& & \mathcal{T}cba \arrow[dll, bend right, "\mathrm{id}"'] \arrow[drr, bend left, "\mathrm{id}"] \arrow[d, "\mathrm{id}\times I^b\times\mathrm{id}"] & & \\
{} \arrow[r, phantom, "\overset{(\rho^{bc})^*\times\mathrm{id}}{\Longrightarrow}"] & {} & \mathcal{T}cbba \arrow[dll, "C^{bbc}\times\mathrm{id}"] \arrow[drr, "\mathrm{id}\times C^{abb}"'] & {} & {} \arrow[l, phantom, "\overset{\mathrm{id}\times\lambda^{ab}}{\Longrightarrow}"] \\
\mathcal{T}cba \arrow[drr, "C^{abc}"'] & & {} \arrow[d, phantom, "\overset{\alpha^{abbc}}{\Longrightarrow}"] & & \mathcal{T}cba \arrow[dll, "C^{abc}"] \\
& & \mathcal{T}ca & &
\end{tikzcd}
\quad \overset{\mathfrak{B}^{abc}}{\underset{\sim}{\Rrightarrow}} \quad
\begin{tikzcd}[column sep=small]
& & \mathcal{T}cba \arrow[dll, bend right, "\mathrm{id}"'] \arrow[drr, bend left, "\mathrm{id}"] & & \\
& & = & & \\
\mathcal{T}cba \arrow[drr, "C^{abc}"'] & & & & \mathcal{T}cba \arrow[dll, "C^{abc}"] \\
& & \mathcal{T}ca & &
\end{tikzcd}
$$

$$
\begin{tikzcd}
 & & \mathcal{T}cba \arrow[dl, "I^c\times\mathrm{id}"'] \arrow[dr, "C^{abc}"] \arrow[ddll, "\mathrm{id}"] & \\
 & \mathcal{T}ccba \arrow[dl, "C^{bcc}\times\mathrm{id}"'] \arrow[r, phantom, "\overset{\lambda^{ac}}{\Longrightarrow}"] & & \mathcal{T}ca \arrow[ddll, "\mathrm{id}"] \\
 \mathcal{T}cba \arrow[dr, "C^{abc}"'] & & = & \\
 & \mathcal{T}ca & &
\end{tikzcd}
\quad \overset{\mathfrak{L}^{abc}}{\overset{\sim}{\Longrightarrow}} \quad
\begin{tikzcd}
 & & \mathcal{T}cba \arrow[dl, "I^c\times\mathrm{id}"'] \arrow[dr, "C^{abc}"] & \\
 & \mathcal{T}ccba \arrow[dl, "C^{bcc}\times\mathrm{id}"'] \arrow[dr, "\mathrm{id}\times C^{abc}"] \arrow[rr, phantom, "="] & & \mathcal{T}ca \arrow[dl, "I^c"] \arrow[ddll, "\mathrm{id}"] \arrow[dd, phantom, "\overset{\lambda^{ac}}{\Longrightarrow}"] \\
 \mathcal{T}cba \arrow[dr, "C^{abc}"'] \arrow[rr, phantom, "\underset{\alpha^{abcc}}{\Longrightarrow}"] & & \mathcal{T}cca \arrow[dl, "C^{acc}"] & \\
 & \mathcal{T}ca & &
\end{tikzcd}
$$

$$
\begin{tikzcd}
 & \mathcal{T}cba \arrow[dl, "C^{abc}"'] \arrow[dr, "\mathrm{id}\times I^a"] \arrow[ddrr, "\mathrm{id}"'] & & \\
 \mathcal{T}ca \arrow[ddrr, "\mathrm{id}"'] & & \mathcal{T}cbaa \arrow[dr, "\mathrm{id}\times C^{aac}"] \arrow[d, phantom, "\overset{\rho^{ac}}{\Longrightarrow}"] & \\
 & = & & \mathcal{T}cba \arrow[dl, "C^{abc}"] \\
 & & \mathcal{T}ca &
\end{tikzcd}
\quad \overset{\mathfrak{R}^{abc}}{\overset{\sim}{\Longrightarrow}} \quad
\begin{tikzcd}
 & \mathcal{T}cba \arrow[dl, "C^{abc}"'] \arrow[dr, "\mathrm{id}\times I^a"] & & \\
 \mathcal{T}ca \arrow[dr, "I^a"'] \arrow[ddrr, "\mathrm{id}"'] \arrow[rr, phantom, "="] & & \mathcal{T}cbaa \arrow[dl, "C^{abc}\times\mathrm{id}"'] \arrow[dr, "\mathrm{id}\times C^{aac}"] & \\
 \overset{\rho^{ac}}{\Longrightarrow} & \mathcal{T}caa \arrow[dr, "C^{aac}"'] \arrow[rr, phantom, "\underset{\alpha^{aabc}}{\Longrightarrow}"] & & \mathcal{T}cba \arrow[dl, "C^{abc}"] \\
 & & \mathcal{T}ca &
\end{tikzcd}
$$

(9) 任意の $a \xrightarrow{f} b \xrightarrow{g} c \xrightarrow{h} d \xrightarrow{j} e \xrightarrow{k} x$ に対して次の等式が成り立つ.

$$
\begin{tikzcd}
 & & & (k(j(hg)))f \arrow[dr] & & \\
 & (k((jh)g))f \arrow[urr] \arrow[drr] & & \Downarrow\wr & k((j(hg))f) \arrow[ddr] & \\
 & & \Downarrow\mathfrak{A} & k(((jh)g)f) \arrow[ur] \arrow[ddr] & & \\
 ((k(jh))g)f \arrow[uur] \arrow[drr] & & & & \Downarrow\mathfrak{A} & k(j((hg)f)) \arrow[dd] \\
 & & (k(jh))(gf) \arrow[rr] & & k((jh)(gf)) \arrow[dr] & \\
 (((kj)h)g)f \arrow[uu] \arrow[drr] & \Downarrow\wr & & \Downarrow\mathfrak{A} & & k(j(h(gf))) \\
 & & ((kj)h)(gf) \arrow[uu] \arrow[rr] & & (kj)(h(gf)) \arrow[ur] &
\end{tikzcd}
$$

$$\parallel$$

$$
\begin{tikzcd}
 & & (k(j(hg)))f \arrow[dr] & & \\
 & (k((jh)g))f \arrow[ur] \arrow[dr, phantom, "\Downarrow\mathfrak{A}"] & & k((j(hg))f) \arrow[dr] \arrow[dd, phantom, "\Downarrow\mathfrak{A}"] & \\
((k(jh))g)f \arrow[ur] & & ((kj)(hg))f \arrow[uu] \arrow[dr] \arrow[ddd, phantom, "\Downarrow\mathfrak{A}"] & & k(j((hg)f)) \arrow[dd] \\
 & & & (kj)((hg)f) \arrow[ur] \arrow[dd] \arrow[dr, phantom, "\Downarrow\wr"] & \\
(((kj)h)g)f \arrow[uu] \arrow[uurr] \arrow[drr] & & & & k(j(h(gf))) \\
 & & ((kj)h)(gf) \arrow[r] & (kj)(h(gf)) \arrow[ur] &
\end{tikzcd}
$$

(10) 任意の $a \xrightarrow{f} b \xrightarrow{g} c \xrightarrow{h} c$ に対して次の等式が成り立つ.

$$
\begin{tikzcd}
 & & (h(Ig))f \arrow[d] \arrow[dr] & \\
 & ((hI)g)f \arrow[ur] \arrow[d] \arrow[dr, phantom, "\Downarrow\mathfrak{A}"] & h((Ig)f) \arrow[dd] \arrow[dddr] & (hg)f \arrow[ddd] \arrow[dl, phantom, "\Downarrow\wr"] \\
(hg)f \arrow[ur] \arrow[ddr] \arrow[r, phantom, "\Downarrow\wr"] & (hI)(gf) \arrow[dr] & & \\
 & & h(I(gf)) \arrow[dr] \arrow[ur, phantom, "\Downarrow\mathfrak{L}"] & \\
 & h(gf) \arrow[uu] \arrow[rr] \arrow[ur, phantom, "\Downarrow\mathfrak{B}"] & & h(gf)
\end{tikzcd}
$$

$$\parallel$$

$$
\begin{tikzcd}
 & & (h(Ig))f \arrow[dr] \arrow[d, phantom, "\Downarrow\mathfrak{B}"] & \\
 & ((hI)g)f \arrow[ur] & {} & (hg)f \arrow[ddd] \arrow[dddll, phantom, "\Downarrow\wr"] \\
(hg)f \arrow[ur] \arrow[urrr] \arrow[ddr] & & & \\
 & & & \\
 & h(gf) \arrow[rr] & & h(gf)
\end{tikzcd}
$$

$$
\begin{tikzcd}
 & h((gI)f) \arrow[dr] & & \\
h(gf) \arrow[ur] \arrow[r, phantom, "\Downarrow\wr"] & (h(gI))f \arrow[u] \arrow[r, phantom, "\Downarrow\mathfrak{A}"] & h(g(If)) \arrow[dr] & \\
 & \arrow[r, phantom, "\Downarrow\mathfrak{R}"] & (hg)(If) \arrow[u] \arrow[dd] \arrow[r, phantom, "\Downarrow\wr"] & h(gf) \\
 & ((hg)I)f \arrow[uu] \arrow[ur] \arrow[r, phantom, "\Downarrow\mathfrak{B}"] & & \\
(hg)f \arrow[uuu] \arrow[uuur] \arrow[ur] \arrow[rr] & & (hg)f \arrow[uur] &
\end{tikzcd}
$$

$$\|$$

$$
\begin{tikzcd}
 & h((gI)f) \arrow[dr] & & \\
h(gf) \arrow[ur] \arrow[drrr] \arrow[rr, phantom, "\Downarrow\mathfrak{B}"] & & h(g(If)) \arrow[dr] & \\
 & \Downarrow\wr & & h(gf) \\
(hg)f \arrow[uu] \arrow[rr] & & (hg)f \arrow[ur] &
\end{tikzcd}
$$

定義. $\mathcal{T},\mathcal{S}$ を tricategory とする．trihomomorphism $F\colon \mathcal{T} \longrightarrow \mathcal{S}$ とは以下を満たすことである．

(1) 関数 $F\colon \mathrm{Ob}(\mathcal{T}) \longrightarrow \mathrm{Ob}(\mathcal{S})$ が与えられている．

(2) 各対象 $a,b \in \mathcal{T}$ に対して pseudofunctor $F^{ab}\colon \mathcal{T}(a,b) \longrightarrow \mathcal{S}(Fa,Fb)$ が与えられている．

(3) 各対象 $a,b,c \in \mathcal{T}$ に対して次の adjoint equivalence φ^{abc} が与えられている．

$$
\begin{tikzcd}
 & \mathcal{T}(b,c) \times \mathcal{T}(a,b) \arrow[dl, "F^{bc}\times F^{ab}"'] \arrow[dr, "C^{abc}"] & \\
\mathcal{S}(Fb,Fc) \times \mathcal{S}(Fa,Fb) \arrow[dr, "C^{FaFbFc}"'] \arrow[rr, phantom, "\overset{\sim}{\Longrightarrow}_{\varphi^{abc}}"] & & \mathcal{T}(a,c) \arrow[dl, "F^{ac}"] \\
 & \mathcal{S}(Fa,Fc) &
\end{tikzcd}
$$

(4) 各対象 $a \in \mathcal{T}$ に対して次の adjoint equivalence ψ^a が与えられている.

$$
\begin{tikzcd}
\mathbf{1} \arrow[rr, "I^{Fa}"] \arrow[dr, "I^a"'] & \arrow[d, phantom, "\psi^a \Downarrow"] & \mathcal{S}(Fa, Fa) \\
& \mathcal{T}(a, a) \arrow[ur, "F^{aa}"'] &
\end{tikzcd}
$$

(5) 各対象 $a, b, c, d \in \mathcal{T}$ に対して次の同型な modification $\mathfrak{F}^{abcd}$ が与えられている.

$$
\begin{tikzcd}
& \mathcal{T}dcba \arrow[rr, "F^{cd} \times F^{bc} \times F^{ab}"] \arrow[dl, "\mathrm{id} \times C^{abc}"'] \arrow[dr, "C^{bcd} \times \mathrm{id}"] & \arrow[d, phantom, "\varphi^{bcd} \times \mathrm{id} \, \Lleftarrow"] & \mathcal{S}dcba \arrow[dr, "C^{FbFcFd} \times \mathrm{id}"] & \\
\mathcal{T}dca \arrow[dr, "C^{acd}"'] \arrow[r, phantom, "\overset{\alpha^{abcd}}{\Leftarrow}"] & {} & \mathcal{T}dba \arrow[rr, "F^{bd} \times F^{ab}"] \arrow[dl, "C^{abd}"] \arrow[d, phantom, "\varphi^{abd} \, \Lleftarrow"] & & \mathcal{S}dba \arrow[dl, "C^{FaFbFd}"] \\
& \mathcal{T}da \arrow[rr, "F^{ad}"'] & {} & \mathcal{S}da &
\end{tikzcd}
$$

$$
\overset{\mathfrak{F}^{abcd}}{\overset{\sim}{\Rrightarrow}}
$$

$$
\begin{tikzcd}
& \mathcal{T}dcba \arrow[rr, "F^{cd} \times F^{bc} \times F^{ab}"] \arrow[dl, "\mathrm{id} \times C^{abc}"'] & \arrow[d, phantom, "\mathrm{id} \times \varphi^{abc} \, \Lleftarrow"] & \mathcal{S}dcba \arrow[dl, "\mathrm{id} \times C^{bcd}"'] \arrow[dr, "C^{FbFcFd} \times \mathrm{id}"] & \\
\mathcal{T}dca \arrow[rr, "F^{cd} \times F^{ac}"] \arrow[dr, "C^{acd}"'] & \arrow[d, phantom, "\varphi^{acd} \, \Lleftarrow"] & \mathcal{S}dca \arrow[dr, "C^{FaFcFd}"] \arrow[rr, phantom, "\overset{\alpha^{FaFbFcFd}}{\Leftarrow}"] & & \mathcal{S}dba \arrow[dl, "C^{FaFbFd}"] \\
& \mathcal{T}da \arrow[rr, "F^{ad}"'] & {} & \mathcal{S}da &
\end{tikzcd}
$$

(6) 各対象 $a, b \in \mathcal{T}$ に対して次の同型な modification $\mathfrak{G}^{ab}, \mathfrak{H}^{ab}$ が与えられている.

$$
\begin{tikzcd}
& \mathcal{T}ba \arrow[dl, "F^{ab}"'] \arrow[dr, "I^b \times \mathrm{id}"] \arrow[ddrr, bend left, "\mathrm{id}"] & \arrow[dr, phantom, "\overset{\lambda^{ac}}{\Rightarrow}"] & \\
\mathcal{S}ba \arrow[dr, "I^{Fb} \times \mathrm{id}"'] \arrow[rr, phantom, "\overset{\psi^b \times \mathrm{id}}{\Rightarrow}"] & & \mathcal{T}bba \arrow[dl, "F^{bb} \times F^{ab}"'] \arrow[dr, "C^{abb}"] & \\
& \mathcal{S}bba \arrow[dr, "C^{FaFbFb}"'] \arrow[rr, phantom, "\underset{\varphi^{abb}}{\Rightarrow}"] & & \mathcal{T}ba \arrow[dl, "F^{ab}"] \\
& & \mathcal{S}ba &
\end{tikzcd}
\quad
\overset{\mathfrak{G}^{abc}}{\overset{\sim}{\Rrightarrow}}
\quad
\begin{tikzcd}
& \mathcal{T}ba \arrow[dl, "F^{ab}"'] \arrow[ddrr, bend left, "\mathrm{id}"] \arrow[drr, phantom, "="] & & \\
\mathcal{S}ba \arrow[dr, "I^{Fb} \times \mathrm{id}"'] \arrow[ddrr, bend left, "\mathrm{id}"] \arrow[r, phantom, ""] & \arrow[d, phantom, "\overset{\lambda^{ac}}{\Rightarrow}"] & & \\
& \mathcal{S}bba \arrow[dr, "C^{FaFbFb}"'] & & \mathcal{T}ba \arrow[dl, "F^{ab}"] \\
& & \mathcal{S}ba &
\end{tikzcd}
$$

$$
\begin{tikzcd}
 & \mathcal{T}ba \arrow[dl, "F^{ab}"'] \arrow[dr, "\mathrm{id}\times I^{a}"] \arrow[ddrr, "\mathrm{id}"'] & & \\
\mathcal{S}ba \arrow[ddrr, "\mathrm{id}"'] \arrow[rr, phantom, "="] & & \mathcal{T}baa \arrow[dr, "C^{aab}"] \arrow[l, phantom, "\overset{(\rho^{ab})^{*}}{\Longrightarrow}"] & \\
 & & & \mathcal{T}ba \arrow[dl, "F^{ab}"] \\
 & & \mathcal{S}ba &
\end{tikzcd}
\quad \overset{\mathfrak{H}^{abc}}{\overset{\sim}{\Rrightarrow}} \quad
\begin{tikzcd}
 & \mathcal{T}ba \arrow[dl, "F^{ab}"'] \arrow[dr, "\mathrm{id}\times I^{a}"] & & \\
\mathcal{S}ba \arrow[rr, phantom, "\overset{\mathrm{id}\times\psi^{a}}{\Longrightarrow}"] \arrow[dr, "\mathrm{id}\times I^{Fa}"] \arrow[ddrr, "\mathrm{id}"'] & & \mathcal{T}baa \arrow[dl, "F^{ab}\times F^{aa}"] \arrow[dr, "C^{aab}"] & \\
\overset{(\rho^{ab})^{*}}{\Longrightarrow} & \mathcal{S}baa \arrow[dr, "C^{FaFaFb}"'] \arrow[rr, phantom, "\underset{\varphi^{aab}}{\Longrightarrow}"] & & \mathcal{T}ba \arrow[dl, "F^{ab}"] \\
 & & \mathcal{S}ba &
\end{tikzcd}
$$

(7) 任意の $a \xrightarrow{f} b \xrightarrow{g} c \xrightarrow{h} d \xrightarrow{j} e$ に対して次の等式が成り立つ.

$$
\begin{tikzcd}[column sep=small]
 & & F(((jh)g)f) \arrow[drrr] \arrow[d, phantom, "\Downarrow\wr"] & & & & \\
 & F((jh)g)Ff \arrow[ur] \arrow[r] & F(j(hg))Ff \arrow[rrr] \arrow[drrr, phantom, "\Downarrow\mathfrak{F}"] & & & F((j(hg))f) \arrow[d] & \\
 & (F(jh)Fg)Ff \arrow[u] \arrow[dr, phantom, "\Downarrow\mathfrak{F}"] & (FjF(hg))Ff \arrow[u] \arrow[dr] & & FjF((hg)f) \arrow[r] \arrow[ddr] & F(j((hg)f)) \arrow[dr] \arrow[d, phantom, "\Downarrow\wr"] & \\
((Fjh)Fg)Ff \arrow[ur] \arrow[rr] \arrow[dr] \arrow[drr, phantom, "\Downarrow\mathfrak{A}"] & & (Fj(FhFg))Ff \arrow[u] \arrow[d] \arrow[r, phantom, "\Downarrow\wr"] & Fj(F(hg)Ff) \arrow[ur] & & & F(j(h(gf))) \\
 & (FjFh)(FgFf) \arrow[dr] & Fj((FhFg)Ff) \arrow[ur] \arrow[d] \arrow[rrr, phantom, "\Downarrow\mathfrak{F}"] & & & FjF(h(gf)) \arrow[ur] & \\
 & & Fj(Fh(FgFf)) \arrow[rrr] & & & Fj(FhF(gf)) \arrow[u] &
\end{tikzcd}
$$

$$\|$$

$$
\begin{tikzcd}[column sep=small]
 & & F(((jh)g)f) \arrow[rrrrr] \arrow[ddrrr] & & & & & & \\
 & F((jh)g)Ff \arrow[ur] & & & & & & F((j(hg))f) \arrow[dr] & \\
(F(jh)Fg)Ff \arrow[ur] \arrow[drr] & & \arrow[r, phantom, "\Downarrow\mathfrak{F}"] & {} & & F((jh)(gf)) \arrow[drrr] & \Downarrow F\mathfrak{A} & & F(j((hg)f)) \arrow[d] \\
((Fjh)Fg)Ff \arrow[u] \arrow[d] & \Downarrow\wr & F(jh)(FgFf) \arrow[rr] & & F(jh)F(gf) \arrow[ur] & & & & F(j(h(gf))) \\
(FjFh)(FgFf) \arrow[urr] \arrow[rrrr] \arrow[ddrrr] & & \Downarrow\wr & & (FjFh)F(gf) \arrow[u] \arrow[ddrrr] & & \Downarrow\mathfrak{F} & & FjF(h(gf)) \arrow[u] \\
 & & & \Downarrow\wr & & & & & \\
 & & & Fj(Fh(FgFf)) \arrow[rrrr] & & & & Fj(FhF(gf)) \arrow[uur] &
\end{tikzcd}
$$

(8) 任意の $a \xrightarrow{f} b \xrightarrow{g} c$ に対して次の等式が成り立つ.

$$
\begin{tikzcd}[column sep=small]
 & & & & F(gI)Ff \arrow[rr] & & F(g(If)) \arrow[ddrr] & & \\
 & & & F(gI)Ff \arrow[ur] & \Downarrow\mathfrak{F} & & FgF(If) \arrow[u] \arrow[dddddrr] & & \\
F(gf) \arrow[uurrrr] & \Downarrow\wr & & (FgFI)Ff \arrow[u] \arrow[rr] & & Fg(FIFf) \arrow[ur] & & \Downarrow\wr & F(gf) \\
 & & \Downarrow\mathfrak{H} & & \Downarrow\wr & & \Downarrow\mathfrak{G} & & \\
 & & F(gI)Ff \arrow[uur] \arrow[rrr] & & & Fg(IFf) \arrow[uu] \arrow[ddrrr] & & & \\
 & & & & \Downarrow\mathfrak{B} & & & & \\
FgFf \arrow[uuuu] \arrow[uuuuurrr] \arrow[uurr] \arrow[rrrrrrrr] & & & & & & & & FgFf \arrow[uuuu]
\end{tikzcd}
$$

$$\|$$

$$\begin{tikzcd}
& F(gI)Ff \arrow[r] \arrow[d, phantom, "\Downarrow F\mathfrak{B}"] & F(g(If)) \arrow[dr] & \\
F(gf) \arrow[ur] \arrow[rrr] & {} \arrow[d, phantom, "\Downarrow\wr"] & & F(gf) \\
FgFf \arrow[u] \arrow[rrr] & {} & & FgFf \arrow[u]
\end{tikzcd}$$

定義. strict 2-category $\mathcal{A},\mathcal{B}$ の Gray テンソル積 $\mathcal{A}\otimes\mathcal{B}$ とは (略)

定義. 圏 **Cat-Cat** は Gray テンソル積により対称モノイダル閉圏となる．これを **Gray** と書く．

定義. **Gray**-豊穣圏を Gray-category という．

定義. trihomomorphism $F\colon\mathcal{T}\longrightarrow\mathcal{S}$ が triequivalence
$\Longleftrightarrow$ F が triessentially surjective かつ，任意の $a,b\in\mathcal{T}$ に対して F^{ab} が biequivalence.

定理 1. 任意の tricategory はある Gray-category と triequivalence である．

証明. 略 $\square$

参考文献

[1] Nick Gurski, An algebraic theory of tricategories, 2007, `http://gauss.math.yale.edu/~mg622/pubs.html`